Optimal Design and Operation of Heat Exchangers under Milk Fouling

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This article presents a procedure for the simultaneous optimization of the design and operation of heat exchangers under milk fouling. This scheme is based on a highly accurate dynamic model described by integral, partial differential, and algebraic equations. Design and operating parameters determined by the dynamic optimization include the length and diameter of the exchanger, the control policy, and the timing of the key operating steps. An economic objective function is used to account for the important factors related to milk heat treatment, and many operating constraints are imposed. Three heat exchanger configurations are considered to study the effect of fouling on the optimal plant design. Optimization indicates that the cost factor due to the interruption of production is dominant, while an increase in energy consumption due to fouling is not very important. The constant wall temperature configuration proved to be the most economical. The economic impact of different control structures was also explored.

Introduction

Heat transfer is important in industrial processing and heat exchangers are used in most stages of industrial transformations. Heat exchanger fouling is the major unresolved problem in this field and is particularly acute for fluids (such as milk) that contain heat-sensitive components (Taborek et al., 1972). The presence of fouling in heat exchanger equipment represents extra capital, energy, and labor costs to the industrial sector. Specifically, the cost of fouling includes (Garett et al., 1985):

- Increased cost due to oversized or redundant equipment
- Additional downtime for maintenance and repair
- Loss of production
- Cleaning equipment and services
- Waste of energy and heat

In dairy plants, *pasteurization* (a milk heat treatment) is an important process that destroys pathogenic bacteria and is required to safeguard public health and ensure good quality dairy products (Hall, 1976). Fouling in dairy plants is a severe problem, especially compared with fouling in chemical plants. The common practice to mitigate fouling in dairy plants is to implement cleaning-in-place (CIP) operations based on predetermined heating–cleaning cycles. These cycles are empirically developed without any knowledge of the actual fouling state of the system and without accounting for broad economic considerations. Furthermore, the design of the equipment and the control policies are also empirical. This results in additional operating and capital costs and unnecessary interruptions to the operation of the plant. In particular, from a design point of view, if the fouling effect is overestimated, the capital cost of the plant is too high, whereas if it is underestimated the resulting plant requires frequent cleaning and thus has high operating costs. In general, the greater the heat transfer area used to perform a given thermal duty, the less the control action required to maintain the process-stream target temperature under fouling (Garett et al., 1985).

It is clear that the size of the heat exchanger adopted for milk heat treatment should be determined by an economical balance between equipment capital cost and potential benefits arising from enhanced thermal efficiency and reduced cleaning costs (Foumeny and Heggs, 1991). Although fouling may eventually settle down to a constant value, in the dairy industry it is generally so severe that cleaning is necessary before this happens. To cope with the changing fouling resistance, process control is needed. If this control is applied incorrectly, for example having wall temperature that is too

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high or product flow rate that is too low, fouling may become more severe (Fryer, 1989).

The need for optimizing the economics of heat exchangers under fouling has long been recognized. Sandu and Lund (1983) discussed the potential for optimal design and operation of heat exchangers in order to mitigate fouling. They emphasized that the dynamics of fouling and the design and operation of heat exchangers used for pasteurization are strongly related and the key point is to quantify and optimize this relationship. These authors presented three basic procedures to minimize fouling cost: economical suppression of fouling resistance, optimum production–cleaning cycle, and standby heat exchangers. They also noted that practical experience shows that engineers improve the operation of heat exchangers under fouling when they are aware of the transient behavior of fouling and take into account the design and operation parameters influencing its dynamics. Engineers need first to determine and understand the fouling dynamics and then decide whether they can be economically suppressed or not and what optimization procedure is suitable in each case (Sandu and Lund, 1985).

Although there is a considerable amount of experimental and theoretical work on milk fouling (Fryer et al., 1996; Delplace et al., 1994), to our knowledge there has been no previous publication concerning optimal design and operation of heat exchangers under milk fouling using detailed models and taking into account comprehensive economic considerations. The dynamics of milk fouling are complex, making the process optimization a difficult mathematical problem. As an attempt to explore aspects of the problem some authors studied the effect of different operating procedures on milk fouling. Yoon and Lund (1994) presented an experimental comparison of two different operating methods on the temperature profile under milk fouling; (i) control by increasing the flow rate of the heating medium, and (ii) control by increasing the inlet temperature of the heating medium. Jong and Linden (1992) presented a mathematical model defining the optimal operating conditions in heat treatment equipment for milk, assuming an analogy with a cascade of nonisothermal plug flow and isothermal tank reactors. The problem of defining the optimal control policy in order to maintain milk temperature close to its target value was not considered. Furthermore, the whole analysis is based on simulation rather than on formal optimization schemes.

Fryer and Slater (1986) used simplified models to simulate various alternative strategies for the control of heat exchangers subject to reaction fouling. Four different configurations of exchangers and control strategies were considered. Although the work of Fryer and Slater is not based on an optimization procedure, and the design problem is not addressed, useful conclusions were obtained with respect to the effectiveness of different control strategies. Fryer et al. (1988) examined the optimal design of heat exchangers under reaction fouling for uniform wall temperature and fixed outlet fluid temperature. The objective was to minimize the amount of fouling in the system. Beyond the fact that a simple model was used in order to quantify fouling, the problem of defining the optimal operating policies was not considered.

One of the weaknesses of the above approaches is that they are not based on optimization techniques but on simulation results. More importantly, simple and/or aggregated models were used, often neglecting fluid hydrodynamics and other important interactions. Also, the interactions between design and control decisions have been ignored. Finally, an explicit economic objective function has never been used since the objective function (if there was one) was to minimize the amount of fouling.

In this article we present a mathematical approach for addressing the problem of optimal design and operation of heat exchangers under milk fouling. The problem is posed as an optimal control problem, based on control vector parameterization (CVP) using the detailed nonlinear process model presented in our previous work (Georgiadis et al., 1998). The optimization procedure is demonstrated for three different configurations of heat exchanger and a variety of control policies. It results in (i) the optimal switching time from heating to cleaning, (ii) the optimal length and diameter of the tube, and (iii) the optimal control policy (profile of the control variables with respect to time).

In the following section, we describe briefly the mathematical model used to optimize the process and propose an economic objective function. We then proceed to analyze the key issues involved in determining optimal design and operating strategies and describe the solution approach. Optimization results for the three cases being studied are presented in the "Solution Approach" section. Finally, the article ends with some comments and closing remarks.

Mathematical Model

The mathematical model used for the optimization procedure is described in our previous work (Georgiadis et al., 1998). This model comprises a set of integral, partial differential, and differential equations (IPDEs) and describes the following:

- The fouling process based on a complex protein reaction scheme.
- The hydro and thermal behavior of the heat exchanger where both axial and radius domains are taken into account.
- Detailed modeling of the transport phenomena which take place during milk heat treatment.
- Monitoring of fouling dynamic behavior.

The model was simulated for four different heat exchanger configurations and the simulation results were validated against experimental data and current industrial techniques for fouling mitigation. Useful conclusions were obtained concerning the behavior of each configuration and the effect of various process parameters on fouling.

The detailed mathematical modeling of the physical phenomena occurring within the heat exchanger under fouling leads to a mixed set of partial differential and algebraic equations of the general form:

$$F(x, x_z, x_{zz}, \dot{x}, y, y_z, u) = 0 \quad \forall z \in (0, L), t \in (0, T] \quad (1)$$

$$\bar{F}(x, x_z, x_{zz}, y, y_z, u) = 0 \quad \forall z \in (0, L), t \in [0, T] \quad (2)$$

where z represents the spatial position in the exchanger, $\dot{x}$ denotes $\partial x/\partial t$, and F and $\bar{F}$ correspond to differential and algebraic equations, respectively. Note that the distinction between the two types of equations is that $\bar{F}(\cdot)$ do not involve time derivatives $\dot{x}$ and also hold at $t = 0$. On the other hand, $F(\cdot)$ do involve $\dot{x}$, they only hold for $t > 0$, and at $t = 0$ they are replaced by initial or other temporal conditions.

For the sake of notation simplicity, here we assume a singe spatial dimension corresponding to the axial position within the exchanger, that is, $z \in [0, L]$ where L is the bed length. However, the models presented in our previous work also include the radial dimension. The methodology presented here is entirely general and is also applied to the radial domain.

We distinguish two classes of variables in the general model described by Eqs. 1 and 2, namely differential $x(z,t)$ and algebraic $y(z,t)$ variables, depending on whether or not their partial derivatives with respect to time also appear in the equations. The number of differential (algebraic) variables is equal to the number of differential (algebraic) equations. The model also involves spatial derivatives (namely $x_z \equiv \partial x/\partial z$, $x_{zz} \equiv \partial^2 x/\partial z^2$, $y_z \equiv \partial y/\partial z$) arising from convective and dispersive phenomena. It also involves certain parameters u that do not vary with either time or spatial position, such as the friction factor or the molecular volume of the absorbed particles. We note also that the partial differential algebraic equations (PDAEs) system 1 and 2 has been written strictly for the interior of the spatial domain $z \in (0, L)$ and not for the boundary conditions $z = 0$ and $z = L$. The phenomena taking place at the latter are described by the boundary conditions. These are given in more detail in our previous work (Georgiadis et al., 1998).

Comprehensive Objective Function

So far, there are no known assessments of heat exchanger fouling costs in food processing. One of the reasons is that food industries have design and operation features that distinguish their equipment from similar equipment in other industries (Sandu and Lund, 1985). Here we propose a detailed objective function to account for all important economic considerations related to fouling. The cost-objective function (to be minimized) not only has to take into account the cleaning cost (which is generated by fouling) but also the cost incurred due to the interruption of production. This is very important and represents the direct economic impact of fouling on the manufacturing plant. Other operating cost terms, such as the annualized capital and energy cost, are also included in the objective function.

We assume that the pasteurizer is periodically cleaned (cyclic operation). In order to quantify the cost of cleaning a simple way to correlate cleaning time, t_{clean}, with heating time, t_{heat}, is proposed based on simulation of the cleaning operation (see Appendix A). The total *cycle* time is then given by the summation of heating and cleaning time.

Cost of cleaning

There are several cost drivers in the cleaning operation. These drivers are the cost of water, the cost of effluent disposal, the cost of steam for heating the cleaning solution and the cost of NaOH. Bird and Espig (1994) reported the cost coefficients for the above drivers.

Water cost. The total amount of water consumed during the cleaning operation is a function of the tube cross-sectional area and the cleaning time. The cost coefficient is \$0.6854/m^3. The total cost of water over the heating–cleaning *cycle* is given by the following expression:

$$CO_{\text{water}} = \frac{0.1192 \cdot \pi \cdot d^2}{4} \cdot t_{\text{clean}} \quad \$/\text{cycle} \tag{3}$$

Cost of NaOH. The solution is 1% wt. and the cost coefficient is \$0.298/kg. In an analogy to the cost of water the final expression over the cleaning time is:

$$C_{\text{NaOH}} = \frac{0.51 \cdot \pi \cdot d^2}{4} \cdot t_{\text{clean}} \quad \$/\text{cycle} \tag{4}$$

Cost of effluent disposal. The cost coefficient is \$1.355/m^3, and the total cost is:

$$C_{\text{ef}} = \frac{0.235 \cdot \pi \cdot d^2}{4} \cdot t_{\text{clean}} \quad \$/\text{cycle} \tag{5}$$

Cost of steam. The cost is expressed in \$0.0283/MJ and it is assumed that steam is used to heat the cleaning solution to the desired temperature (here 60°C). The total cost is given by:

$$C_{\text{steam}} = \frac{0.824 \cdot \pi \cdot d^2}{4} \cdot t_{\text{clean}} \quad \$/\text{cycle} \tag{6}$$

In the above expressions the cleaning solution velocity (0.174 m/s) was used for the necessary unit transformations, while the cleaning time can be replaced by the heating time using Eq. A5 in Appendix A. Therefore, the above expressions have as unknowns the heating time and the tube diameter, d (time invariant parameter for the optimization). The total cleaning cost is given by the following equation:

$$C_{\text{cleaning}} = CO_{\text{water}} + C_{\text{NaOH}} + C_{\text{ef}} + C_{\text{steam}} \tag{7}$$

Cost of heating medium

Three heating configurations are considered: (i) uniform wall temperature, (ii) cocurrent operation with a heating medium, and (iii) countercurrent operation. For the constant wall temperature case it is assumed that the wall temperature is determined by steam available from the utility system in the plant. The milk flow rate, w_{milk}, is constant and equal to 0.25 kg/s for the purpose of our analysis. The amount of steam required is given by the following simple energy balance equation:

$$m_{\text{steam}} = \frac{w_{\text{milk}} \cdot C_{p,\text{milk}} (T_m^{\text{out}} - T_m^{\text{in}})}{\lambda_{\text{steam}}} \quad \text{kg/s} \tag{8}$$

where the enthalpy of vaporization λ_{steam} is a function of its temperature (or tube wall temperature) and is given by:

$$\lambda_{\text{steam}} = 2{,}537 - 2.8013 \cdot T_w \quad \text{J/kg} \tag{9}$$

In this way, the optimization algorithm manipulates the wall temperature T_w as control variable and the corresponding steam cost can be easily calculated. The cost coefficient is taken approximately as \$0.062/kg and the total steam cost over the heating operation is given by the following expression:

$$C_{\text{steam2}} = \int_0^{t_{\text{heat}}} 0.062 \cdot m_{\text{steam}} dt \quad \$/\text{cycle} \tag{10}$$

Note that T_m^{out} and T_m^{in} are the milk outlet and inlet temperature, respectively, and w_{milk} is the mass-flow rate of milk equal to 0.25 kg/s.

For the cocurrent and countercurrent operation the energy cost of the heating medium is also taken into account. Since its inlet temperature is a control variable it is assumed that this temperature is determined by steam from the utility system using an auxiliary exchanger. In order to be consistent with the economic analysis of the constant wall temperature case, the required amount of steam is calculated by a similar balance as follows:

$$m_{\text{steam}} = \frac{w_s \cdot C_{ps} \cdot (T_s^{\text{in}} - T_s^{\text{out}})}{\lambda_{\text{steam}}} \quad \text{kg/s} \tag{11}$$

where T_s^{in} and T_s^{out} are the inlet and outlet heating medium temperatures, respectively, and w_s is the flow rate. It is assumed that the heating medium is circulated between the auxiliary exchanger and the pasteurizer. The total cost over one heating cycle is given by an expression similar to Eq. 10.

Heating medium pumping cost

The pumping cost of the heating medium (for the cocurrent and countercurrent operation), due to frictional pressure drop, must also be included in the objective. This cost is a function of the heating medium flow rate. After suitable unit transformations the following expression is added in the objective function to account for the pumping cost:

$$C_{\text{pump}} = \Delta P_s \cdot C_{pp} \cdot w_s \cdot 1.8 \times 10^{-8} \quad \$/\text{s} \tag{12}$$

where C_{pp} is the cost coefficient equal to \$0.15/kWh and ΔP_s is the pressure drop calculated using standard models (Peters and Timmerhaus, 1981). The pumping cost can then be easily calculated over the period of one heating cycle as shown for the other cost factors.

Cost of heat exchanger

The capital cost of the heat exchanger is represented by a depreciation rate over a one-year period using a suitable charge factor (Douglas, 1988). The capital cost of a heat exchanger is a function of its material and heat-transfer area. Here it is assumed that the construction material for the shell and tube is stainless steel. The approximate capital cost is given by:

$$\text{CHEN} = 2{,}403[10.75 \cdot \text{Area}]^{0.65} \quad \$$$
$$\text{Area} = \pi \cdot d \cdot L$$

where L is the heat exchanger length. The above cost equation is multiplied by the capital charge factor 1/3 (Douglas, 1988) and the final expression is as follows:

$$\text{CHEN} = 801[10.75 \cdot \text{Area}]^{0.65} \quad \$/\text{yr} \tag{13}$$

Cost due to interruption of production

Another very important cost factor which must be added in the objective function is the interruption of production during the cleaning operation (heat exchanger downtime). The model must penalize system downtime because this disruption results in considerable loss of production.

The above can be illustrated by the following. Consider the case where the duration of the heating operation is 1 h. The corresponding time for the required cleaning will be 0.365 h. Over a period of 1 year the total number of cycles (heating + cleaning) will be 5,455 while the total duration of the heating period is 5,455 h. On the other hand, if the heating time is 55.5 h the cleaning time is 0.617 h and the total heating period over one year is 7,364 h. In order to take into account the cost of disruption, the cost model proposed by Mohamed (1996) is modified for the problem considered here. Thus, production losses can be represented by the following cost equation:

$$C_{\text{losses}} = w_{\text{milk}} \cdot S_{\text{net}} \cdot t_{\text{clean}} \quad \$/\text{cycle} \tag{14}$$

where S_{net} is a sales value that represents the net profit per unit of product. This value can be easily determined and for the purposes of our analysis is taken as \$0.25/kg. The above cost is expressed in dollars per cycle.

The approach adopted provides results equivalent to those that would have been obtained by maximizing the net present value (NPV) of the design. This is because the production losses reduce the annual revenue per pasteurizer and therefore the NPV of the project. The adoption of a production loss term enables us to pose the problem as a cost-minimization problem.

Summary of objective function

The objective function that represents the total cost over one year, including production loss, is given by the following expression:

$$CC = \text{NCL} \int_0^{t_{\text{heat}}} [C_{\text{cleaning}} + C_{\text{steam2}} + C_{\text{losses}} + C_{\text{pump}}] dt + C_{HE} \quad \$/\text{yr} \tag{15}$$

where the total number of cycles per year, NCL, is:

$$\text{NCL} = \frac{7{,}446}{t_{\text{heat}} + t_{\text{clean}}} = \frac{7{,}446}{0.353 + 1.01237 \cdot t_{\text{heat}} - 2.55 \times 10^{-4} \cdot t_{\text{heat}}^2 + 1.87 \times 10^{-6} \cdot t_{\text{heat}}^3}$$

where 7,446 is the total working hours of the plant over one year. Cleaning time, t_{clean}, has been replaced by Eq. A5 in the Appendix, leaving as unknown the heating time, t_{heat}.

Minimization of this objective function clearly imposes a penalty due to the interruption of production and necessary cleaning operations.

Optimization Problem

The optimization procedures proposed determine: (i) the optimal heating time of the exchanger (or switching time from heating to cleaning), (ii) the optimal control profiles (time variation of either the wall temperature or the inlet heating medium temperature and flow rate) and, *simultaneously*, (iii) the optimal design (length and diameter) of the tube. The complexity of the dynamic optimization problem arises primarily from the distributed and highly nonlinear nature of the system model.

There are two main issues which must be taken into consideration when establishing optimal control strategies for this problem. The first issue is to ensure that the milk outlet temperature is always kept close to its target value (within the capabilities of the hot utilities in the plant). The second issue is to seek the best economic performance. However, because of the complexity of the underlying physical process, it is often difficult to define simple heuristic strategies in order to address these two issues and take into account all the operating constraints.

Optimization decision variables

In this study, three heat exchanger configurations are optimized: (i) constant wall temperature using steam as the heating medium, (ii) cocurrent operation of milk with any available heating medium (such as hot oil or water), and (iii) the same as in case (ii) but in countercurrent operation.

For the constant wall temperature configuration, the only available control variable is the wall temperature. This is assumed to be determined by steam available in the plant that enters and exits the tube at the same temperature. In a processing site, steam is usually available at a set of pressure conditions (and therefore a set of corresponding temperatures). However, for the purpose of our analysis steam is assumed to be available at any temperature below a predefined upper limit which represents its availability in the plant. For the cocurrent and countercurrent operation both heating medium flow rate and inlet temperature are control variables which are also subject to similar upper bounds.

In pasteurization the selection of the residence time, which is a function of the flow rate and the length and diameter of the tube, is an important issue. In our analysis, the residence time is implicitly optimized through the length and diameter of the tube. In general, milk flow rate (which is constant) may also be an optimization variable.

Spatial domain normalization

A complication that arises in the context of optimization is that the lengths of both the axial and radial domains are decision variables and these appear only implicitly in many of the equations determining their domain of definition (e.g., $z \in [0, L]$). We therefore define a normalized space coordinate $\zeta \in [0,1]$ as follows:

$$\zeta \equiv \frac{z}{L} \qquad (16)$$

All partial derivatives and integrals can then be expressed in terms of this new independent variable:

$$\frac{\partial}{\partial z} = \frac{1}{L}\frac{\partial}{\partial \zeta} \qquad (17)$$

$$\frac{\partial^2}{\partial z^2} = \frac{1}{L^2}\frac{\partial^2}{\partial \zeta^2} \qquad (18)$$

Following these normalizations, the general model Eqs. 1 and 2 become:

$$F\left(x, \frac{1}{L}x_\zeta, \frac{1}{L^2}x_{\zeta\zeta}, y, \dot{x}, \frac{1}{L}y_\zeta, u\right) = 0 \quad \forall \zeta \in (0,1) \quad (19)$$

$$\bar{F}\left(x, \frac{1}{L}x_\zeta, \frac{1}{L^2}x_{\zeta\zeta}, y, \frac{1}{L}y_\zeta, u\right) = 0 \quad \forall \zeta \in (0,1) \quad (20)$$

Similar transformations are applied to the boundary conditions. A new normalized independent variable is also introduced for the radial domain. We note that all the transformed equations are now expressed over *fixed* domains of unit length while the quantity L, and similarly the quantity R for the radius, appear explicitly and can therefore be subject to optimization.

Constraints

Some constraints must be satisfied at all times during the heating operation and are known as "path" constraints. Here there are three types of path constraints. The first concerns the average pressure drop at the milk side which must be less than an upper bound. Following Kern (1965) this bound is set to 7 psi (48 kPa). The second type is the pressure drop at the heating medium side which must also be less than 7 psi (48 kPa). Finally, milk outlet temperature must be maintained close to its target value. A lower bound of 369.5°C is assumed. Note here that no upper bound in the exit temperature is required, since in order to minimize the energy consumption the optimization algorithm will enforce this temperature close to the lower bound. These constraints can be mathematically stated as follows:

$$-\Delta P(t) \leq 7 \text{ psi } \forall t \in [0, t_{\text{heat}}] \qquad (21)$$

$$-\Delta P_s(t) \leq 7 \text{ psi } \forall t \in [0, t_{\text{heat}}] \qquad (22)$$

$$369.5 \leq T_m^{\text{out}}(t) \ \forall t \in [0, t_{\text{heat}}] \qquad (23)$$

Other typical constraints considered include upper and lower values on the time invariant parameters (tube length and diameter). The upper bound on the heating period (time horizon) is assumed to be defined by microbiological constraints and equal to 70 h. Specific attention should be given also to the control variables T_w, T_s^{in}, and w_s to which an upper bound is imposed (derived from the plant data). Finally, the lower bound in the tube diameter, 0.018 m, is defined by pressure drop limitations. The following inequalities express these constraints:

$$0.018 \leq d \leq 0.030 \text{ m} \qquad (24)$$

$$1 \leq L \leq 20 \text{ m} \qquad (25)$$

$$374 \leq T_w \leq 390 \text{ K constant wall temperature case} \quad (26)$$

$$380 \leq T_s^{in} \leq 400 \text{ K cocurrent or countercurrent operation} \quad (27)$$

$$0.1 \leq w_s \leq 10 \text{ kg/s cocurrent or countercurrent operation} \quad (28)$$

$$3 \leq t_{heat} \leq 70 \text{ h} \quad (29)$$

The optimal control problem can then be formulated mathematically as follows:

$$\min_{t_{heat}, T_w(t), T_s^{in}(t), w_s(t), \text{diam}, L, t, \in [0, t_{heat}]} CC(t_{heat})$$

subject to

- $CC(t_{heat})$ as given by Eq. 15.
- The heat exchanger dynamic model under milk fouling, given by the equations, initial and boundary conditions present in our previous work (Georgiadis et al., 1998).
- Control variable bounds: either constraint 26 for the constant wall temperature configuration, or constraints 27 and 28 for cocurrent or countercurrent operation.
- Time horizon bounds, constraint 29.
- Path constraints 21 to 23.
- Design parameter bounds, constraints 24 and 25.

Solution Approach

An important feature of the optimization problem defined in the previous section is that the model equations depend on time t, axial and radial position. As in our previous work (Georgiadis et al., 1998), we apply a spatial discretization method to eliminate the last two independent variables, leading to an optimization problem described by differential algebraic equations only. Nonetheless, it remains a complex, infinite dimensional, nonlinear optimization problem. The infinite dimensionality arises because the state and control variables (for instance, milk temperature and heating medium inlet temperature) are functions of time rather than simple scalar quantities. Several numerical methods exist to solve such dynamic optimal control problems. A class of methods that is widely used, especially in chemical engineering applications, is control vector parameterization (Vassiliadis, 1994a, b). This approach converts the infinite dimensional optimization problem into a finite dimensional one and allows the use of standard nonlinear programming (NLP) solution techniques for the solution of the finite-dimensional problem. The reduction in dimensionality is performed by using a piecewise Lagrange polynomial to approximate the control vector. This allows the control variables to be approximated by polynomials of various degrees and orders of continuity such as piecewise constant, piecewise linear, or quadratic functions.

Assume that $u(t)$ is a control variable (e.g., the wall temperature T_w). To make the dynamic optimization problem tractable the entire time horizon t_{heat} is first divided into N intervals with lengths $l_1, \ldots l_N$:

$$t_{heat} = \sum_{k=1}^{N} l_k \quad (30)$$

The values of l_k are not necessarily fixed or equal to each other. The control actions over each time interval are represented by:

$$u(t) = \Phi_k(t, \upsilon_k) \quad t \in [l_{k-1}, l_k], \quad k = 1, \ldots, N \quad (31)$$

where Φ_k are simple polynomial functions of time t (e.g., constant, linear, or quadratic). In some cases additional constraints are imposed on the continuity of the values of the control variable at the boundaries of the intervals, or even their derivatives with respect to time. For example, if we consider piecewise linear control profiles for the control variable T_w then the following expressions are added in the model

$$\frac{\partial T_w(t)}{\partial t} = \alpha \quad (32)$$

$$T_w(t) = 374 \quad t = 0 \quad (33)$$

where α is a parameter determined by the optimization algorithm.

The precise form of Φ is determined by the finite set of parameters υ and l. It is important to note that if an optimization procedure determines the values for υ and l, it will also have determined as a result the optimal control profile u within the space of the allowable functions. It is also important to emphasize that in many cases the choice of the form of the control variable is an engineering rather than a mathematical issue. It depends very much on the actual control strategy that one can or is willing to implement in practice.

The number of intervals N depends on the practical problem to be solved. Thus, the parameterized control variable u in interval k at time t is given by:

$$u_k(t) = u_k \quad k = 1, \ldots, N$$

In summary, the duration of each time interval and the parameterized control variable u_k are optimization decision variables with prespecified bounds:

$$u_k^{min} \leq u_k \leq u_k^{max} \quad k = 1, \ldots, N$$

$$l_k^{min} \leq l_k \leq l_k^{max} \quad k = 1, \ldots, N$$

The bounds l_k^{min} are set to a small value to avoid numerical problems that may occur if an interval k collapses to zero duration. The upper bounds u_k^{max} reflect the limitations of the hot utilities in the plant (e.g., maximum available steam pressure).

The aim of the optimization is to determine the quantities u_k, l_k, t_{heat}, d and L so as to minimize the objective function while satisfying the constraints. Note here that both the design and control problems are simultaneously addressed.

So far, the control variable in the original problem has been reduced to a finite dimensional representation through control vector parameterization. However, the process model equations are still infinitely dimensional as they have to be enforced at all times $t \in [0, t_{heat}]$. To deal with this problem we note that if u_k, l_k, d, L, and t_{heat} are given, the model equations with their initial conditions determine both the ini-

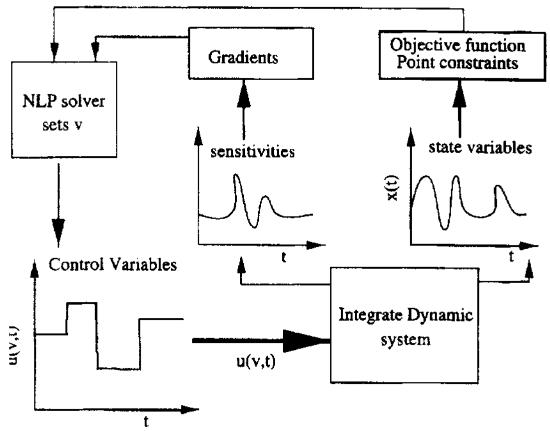

Figure 1. Algorithm of dynamic optimization.

tial solution of the system and its subsequent behavior for $t \in [0, t_{\text{heat}}]$. Once the final values of all the variables are determined in this fashion, then simply the values of the objective function and constraints can be determined. Overall the infinite dimensional optimization problem in the optimization problem section is now reduced to a finite dimensional problem which has a nonlinear programming (NLP) form. For the path constraints and in order to avoid numerical difficulties, a small cumulative violation is assumed over the entire time horizon and a special way to handle them is followed (Vassiliadis et al., 1994b).

The dynamic optimization problem presented in the "Mathematical Model" section is solved using *gOPT*, an implementation of the CVP approach in the *gPROMS* modeling system (Vassiliadis, 1994a,b; *gPROMS* project team, 1996). The normalized axial domain is discretized using second order orthogonal collocation on 10 finite elements, whereas the normalized radial domain is approximated by third order orthogonal collocation on four elements. The equations that describe the model are generated by *gPROMS* as residual equations with symbolically generated partial derivatives (Jacobian) and used as inputs to *gOPT*. The latter employs:

• A sophisticated integrator, DASOLV (Jarvis and Pantelides, 1992) based on backward differentiation formulae methods for integrating the DAEs.

• The SRQPD nonlinear programming code (Chen and Macchietto, 1988) implementing a reduced sequential quadratic programming algorithm. The NLP algorithm requires the gradients of the constraints and the objective function with respect to the optimization decision variables. These can be computed from the "sensitivities" of the state variables with respect to the optimization parameters. The solution of the optimization problem is a two-stage iterative procedure in *gOPT*. The first stage is the integration of the discretized model to obtain constraint residuals and/or gradients, and the second stage is the input of this information to SRQPD. Given initial estimates for optimization variables $p \equiv (d, L, t_{\text{heat}}, u_k, l_k, k = 1, \ldots, N)$, repeat the following two steps until convergence.

1. Integrate DAEs to determine the objective function and constraints and if required their gradients with respect to p.
2. Call SRQPD to determine new estimates for p.

The above algorithm is shown in Figure 1. Initial points along with lower and upper bounds of all optimization variables are given in Table 1.

Table 1. Bounds and Initial Points for the Optimization Variables

Optimization Variable	Lower Bound	Upper Bound	Initial Values
Control Intervals (s)	200	30,000	9,000
Diameter, m	0.018	0.030	0.025
Length, m	1	20	8

Results and Discussion

The effect of time intervals and type of control profiles on the objective function is investigated for the three configurations being optimized. Two types of controls are considered: piecewise constant and piecewise linear controls.

Constant wall temperature case

The results obtained for both the piecewise constant and linear profiles are shown in Table 2. Note here that the control values for the piecewise linear case are the optimal values of parameter α in Eq. 32.

The number of control intervals has a considerable impact on the objective function and four control intervals are not enough to fully optimize the problem. This is clearly shown in the case when eight intervals are used, resulting in a 10% improvement in the objective function. Finally, using 12 intervals the total improvement (compared with the case in which four intervals were used) is about 13% for both types of controls. Piecewise constant controls provide better results than piecewise linear controls when four and eight intervals are used. However, both type of control strategies result in the same value of the objective function for the case of 12 intervals. This is due to the fact that when the number of control intervals is large, piecewise linear profiles can be approximated by piecewise constant profiles. It should be em-

Table 2. Optimization Results for Constant Wall Temperature Case

Type of Control	No. of Control of Intervals	Heating Time (h)	Objective Function ($/yr)	Dia. (m)	Length (m)
Piecewise constant	4	38.7	76,600	0.018	10.7
Piecewise constant	8	52.7	69,000	0.018	10.7
Piecewise constant	12	56.6	67,200	0.018	10.7
Piecewise linear	4	35.7	79,000	0.018	10.7
Piecewise linear	8	44.38	72,000	0.018	10.7
Piecewise linear	12	56.8	67,000	0.018	10.7

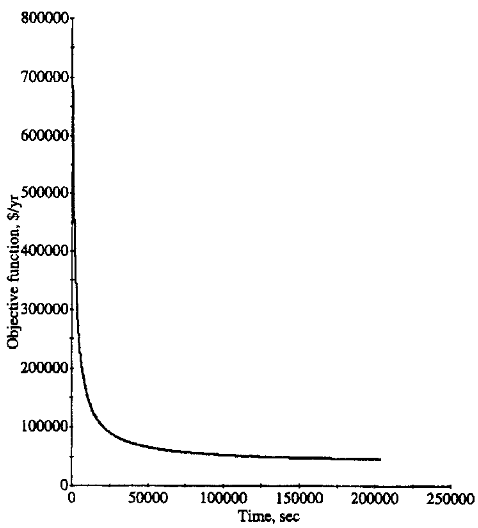

Figure 2. Profile of the objective function.

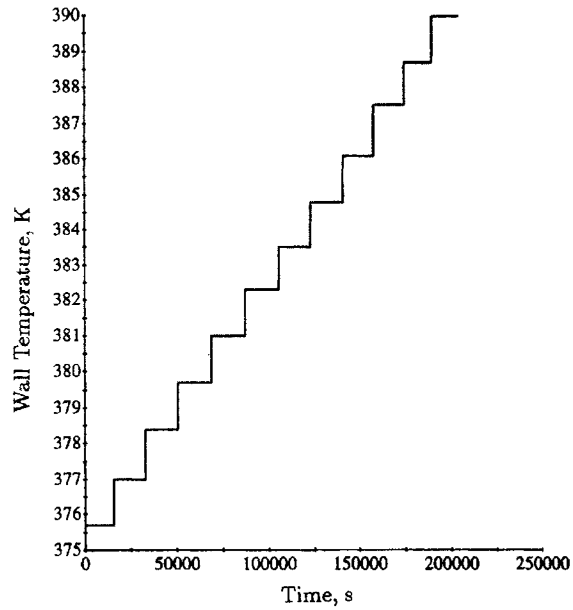

Figure 4. Optimal control policy. Piecewise constant profile, 12 intervals.

phasized here that the cost factor due to interruption of production is dominant and thus the optimization algorithm is set up to minimize the cleaning time. This can be seen in the optimal design parameters and heating times given in Table 2. To illustrate this, a typical behavior of the objective function with respect to heating time for 12 control intervals with piecewise constant control is shown in Figure 2. We can see that the operating cost is inversely proportional to the length of the heating cycle.

Once the optimal design and heating policy are derived, a dynamic simulation was used in order to check the validity of the optimal solution. This is a key step which allows us to check if the process constraints are satisfied, if the results are reasonable according to the physics of the problem, and to confirm any assumption made to simplify the solution procedure. Here, for illustration purposes, only the piecewise constant control profile case with 12 intervals is simulated. The milk outlet temperature as a function of time is shown in Figure 3 and the corresponding control profile is depicted in Figure 4. Clearly, the milk outlet temperature is kept above the lower limit in order to satisfy the temperature requirements of the pasteurization process.

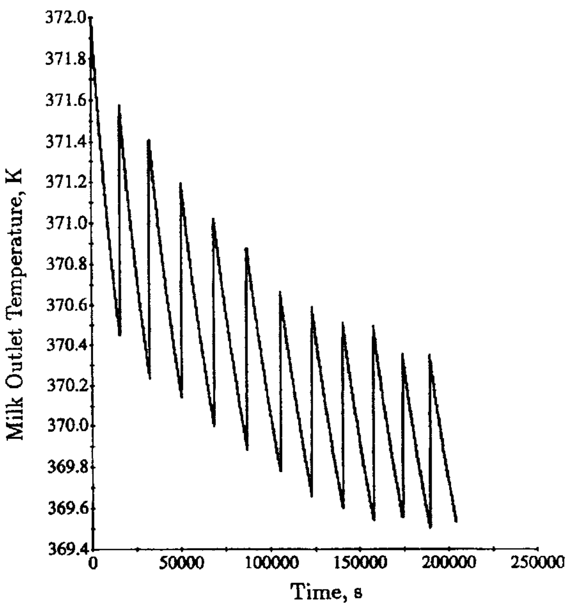

Figure 3. Milk outlet temperature under control for the constant wall temperature case.

Cocurrent operation

For this case, both the heating medium inlet temperature and flow rate are control variables. Here only piecewise constant control profiles are considered. The final results for six and twelve control intervals and the optimal design parameters are shown in Table 3.

Again the results are verified using simulation. The milk outlet temperature profile is shown in Figure 5 and the heating medium inlet temperature and flow rate in Figures 6 and 7. One can see that the milk outlet temperature satisfies the specified bounds over the entire heating period. It is worth noticing, that during some of the time intervals the heating medium inlet temperature decreases while the flow rate increases. On the one hand, the algorithm is trying to minimize the energy consumption (keep heating medium inlet temperature as low as possible), while the energy supply must increase to mitigate fouling effects. It is then possible for the inlet temperature to decrease if fouling can be mitigated by the proper manipulation of flow rate increase. It should also be noted that the observed "spike" in Figures 6 and 7 is due to the small values of the lower bound of the control intervals (200 s).

The case of having only the heating medium inlet temperature as a control variable is also investigated (flow rate is kept on its nominal value). The results for piecewise constant

Table 3. Optimization Results for Cocurrent Operation

Type of Control	No. of Control Intervals	Heating Time (h)	Objective Function ($/yr)	Dia. (m)	Length (m)
Piecewise constant	6	26.1	87,500	0.018	11.8
Piecewise constant	12	24.8	85,200	0.018	11.8

control profile and 12 intervals are shown in Table 4. It is worth noticing that this control scheme is not economical (100% increase in the objective function) compared with the previous one. This is because the switching time to cleaning is significantly shorter. The case of having only the flow rate as control variable was also considered. Here, it was not possible to find even a feasible solution, and both the optimization and the integration/sensitivities stage were infeasible. This is due to the fact that the heating medium flow rate on its own is not an effective variable for control purposes.

Countercurrent operation

Optimization results for piecewise constant control profiles with six and twelve intervals are summarized in Table 5. Again, the use of the heating medium inlet temperature as the only control variable is investigated leading to higher values of the objective function compared with the previous scenario (Table 6). The use of flow rate as the only control variable was proved to be infeasible.

An accurate simulation at the optimal solution is necessary for verification purposes. The milk outlet temperature and the control profiles are depicted in Figures 8 to 10.

Sensitivity analysis of the optimization results

Sensitivity analysis is often useful in dynamic optimization problems to identify the sensitivity of the objective function with respect to some key parameters. The process model includes a number of parameters that are subject to uncertainty. This is especially true for parameters which are determined by experimental data, such as the activation energies E_N and E_D in the fouling reaction scheme, and for the constant β used to quantify the amount of fouling. Moreover, the initial milk protein concentration $C_{N,\mathrm{in}}$ is also an uncertain parameter due to the variability of the raw milk concentration. Finally, the profit coefficient S_{net} can also be modified by market changes. In addition to the process parameters summarized above, the sensitivity with respect to design parameters such as tube diameter was also considered. The analytical values of the sensitivities were obtained from the DASOLV integrator (Jarvis and Pantelides, 1992) through the *gPROMS* modeling system.

The results of the sensitivity analysis are summarized in Table 7 and refer to the constant wall temperature case with piecewise constant control profile and eight intervals. It is clear that the objective function is very sensitive to the tube diameter and the S_{net} parameter. The former affects both the heat exchanger capital cost and more importantly the amount of fouling, while the latter represents a considerable part of the objective function. The objective function is also quite sensitive to the milk composition $C_{N,\mathrm{in}}$ and less so to parameter β. Finally it does not seem to change considerably with respect to the reaction's activation energies, E_N and E_D.

Discussion

Based on the optimization results some general conclusions are summarized here:

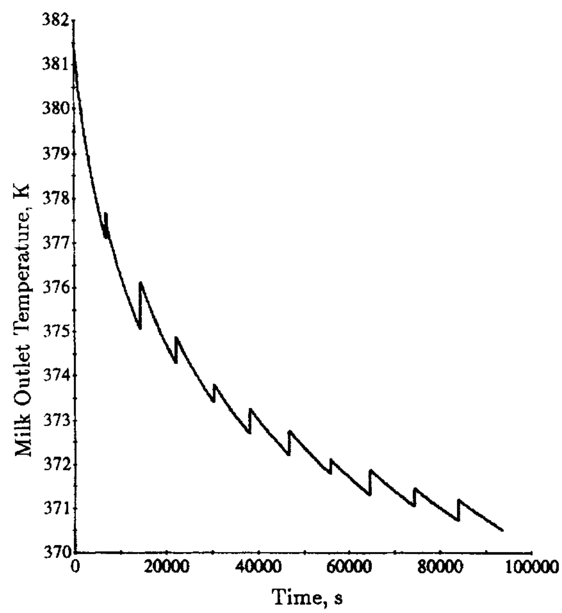

Figure 5. Milk outlet temperature profile under control for cocurrent operation.

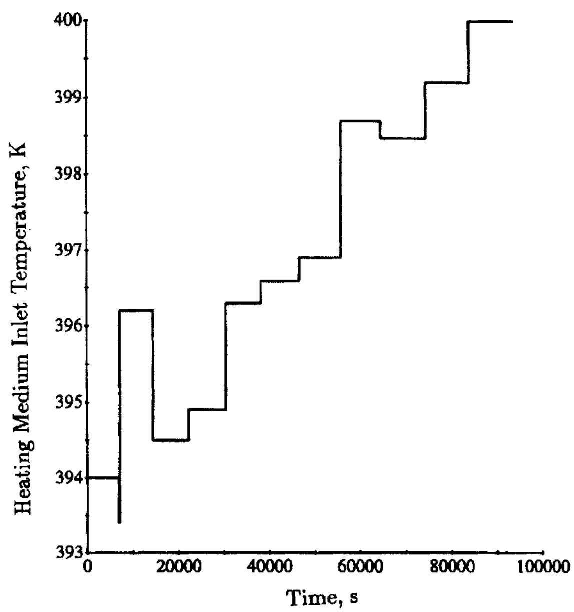

Figure 6. Heating medium inlet temperature profile for the cocurrent operation.

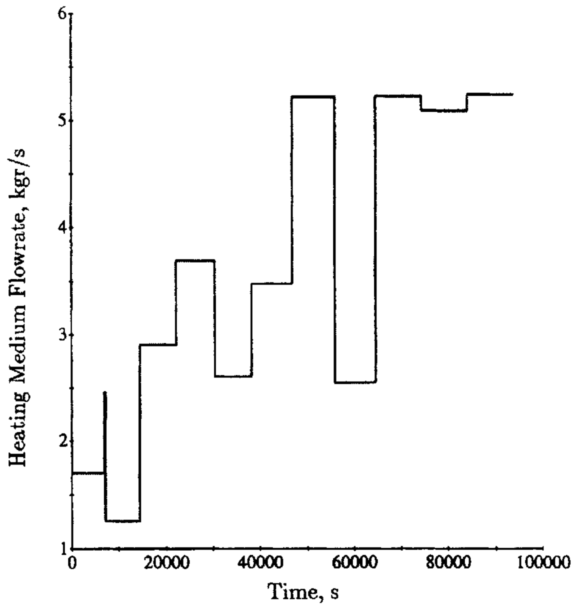

Figure 7. Heating medium flow rate profile for the cocurrent operation.

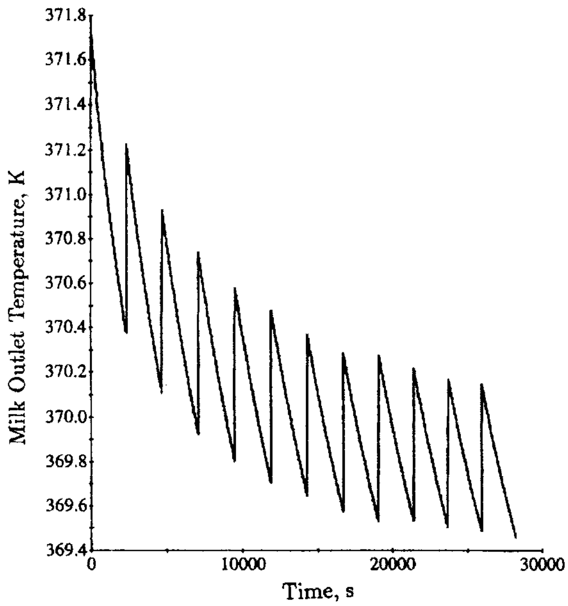

Figure 8. Milk outlet temperature profile under control for the countercurrent operation.

• The interruption of the production has a great effect on the objective function and represents approximately 65% of the total cost. The greater the heating time, the less the annual cost is (Figure 2). It is clear that in all cases the optimization procedure is trying to minimize downtime (maximize heating–cleaning cycle) under the specified constraints. It is worth noticing that in the work of Sandu and Singh (1991) it was emphasized that the energy increase due to heat exchanger fouling was about 8.3% of the total energy consumption and the associated costs represented only about 0.5% of the value of product shipments. Our results support these conclusions. The key issue is to minimize the production losses due to interruption of production during the cleaning operation. Fryer (1989) also posed an important question: Are short but frequent cleanings more efficient than infrequent but lengthy ones? From our results it is clear that lengthy heating cycles are more cost effective but control actions must be optimally chosen since they greatly affect fouling.

• The operability bottleneck in the optimal operation of heat exchangers under fouling is the availability of hot utilities in the plants. It seems that a higher maximum steam temperature would provide a more cost-effective operation. Pressure drop limitation is also a bottleneck for the cocurrent and countercurrent cases of operation.

• The constant wall temperature case results in longer heating times than the other two configurations. This is because milk is heated gradually (with respect to the axial domain) and thus fouling is minimized (see also the analysis in Georgiadis et al., 1998). The countercurrent operation results in shorter heating cycles where milk is heated quite rapidly with high mean temperature difference (countercurrent configurations provide maximum heat recovery) and thus fouling

Table 4. Optimization Results for Cocurrent Operation with One Control Variable

Type of Control	No. of Control Intervals	Heating Time (h)	Objective Function ($/yr)	Dia. (m)	Length (m)
Piecewise constant	12	7.22	166,900	0.018	12.1

Table 5. Optimization Results for the Countercurrent Operation

Type of Control	No. of Control Intervals	Heating Time (h)	Objective Function ($/yr)	Dia. (m)	Length (m)
Piecewise constant	6	6.82	170,000	0.018	13.6
Piecewise constant	12	7.84	158,300	0.018	13.6

Table 6. Optimization Results for the Countercurrent Operation with One Control Variable

Type of Control	No. of Control Intervals	Heating Time (h)	Objective Function ($/yr)	Dia. (m)	Length (m)
Piecewise constant	12	6.3	182,300	0.018	13.9

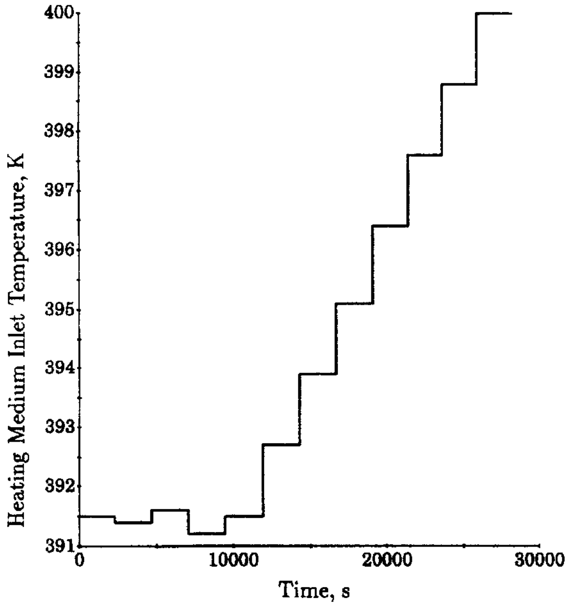

Figure 9. Heating medium inlet temperature profile for the countercurrent operation.

is facilitated. These differences are apparent in Figures 3 and 5. The first scheme maintains milk temperature below 371.8 K while the second raises it up to 381 K.

- The heating medium inlet temperature is not an economic control variable if it is used on its own. The combination with the flow rate results in an improved control scheme. Control only by the flow rate does not seem to be feasible for constant inlet temperature. Similar conclusions were obtained by Yoon and Lund (1994) and Fryer and Slater (1986) in their experimental and simulation work, respectively.
- The optimal tube diameter is relatively small (fixed at its lower bound), since high Reynolds numbers lead to longer

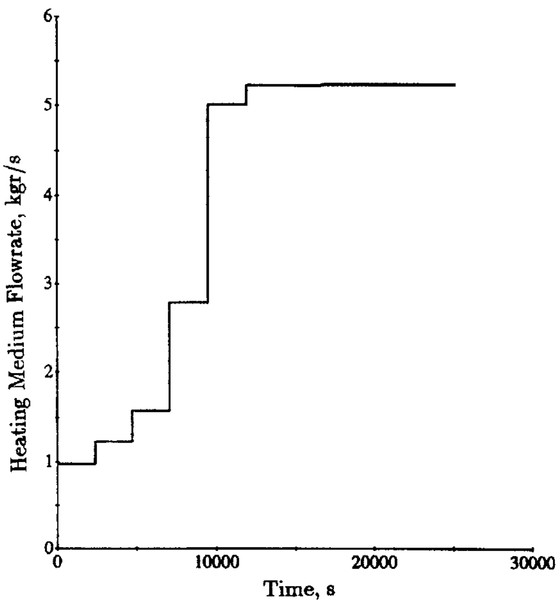

Figure 10. Heating medium flow rate profile for the countercurrent operation.

Table 7. Sensitivity Analysis of the Objective Function for the Constant Wall Temperature Operation

Parameter	Sensitivity of Objective Function, Absolute Values
E_N	0.203
E_D	0.393
$C_{N,\mathrm{in}}$	813.6
β	93.88
Diameter	800,498
S_{net}	960,100

heating cycles. As shown in the simulation results (Georgiadis et al., 1998), substantial mitigation of fouling is achieved by increasing the Reynolds number. This validates current industrial techniques for fouling mitigation. On the other hand, tube length in all cases is large enough to allow milk to be heated gradually. This also verifies current heat exchanger design techniques where most exchangers are oversized to mitigate the undesirable fouling effects. It is worth noticing, however, that the optimal length of the tube is not at its upper bound. This shows that, if larger heat exchangers are to be used, the benefits from the extra fouling mitigation would be counterbalanced by the additional capital expenditure.

In general, the optimization approach presented in this work determines the optimal time programs for the control variables. These time programs can then be implemented in the control system of the plant to define optimal on-line control policies. However, extra work is required for the development of an integrated on-line approach.

Conclusions

This work addresses the optimal design and operation of heat exchangers under milk fouling. The dynamic optimization approach adopted, based on control vector parameterization, identifies simultaneously the optimal design parameters and operating policy of the exchanger. From the results it is clear that the cost factor due to the interruption of production (downtime) is dominant and the heating cycle should be as long as possible. The constant wall temperature configuration provides the best economical results compared with the cocurrent and countercurrent operation. However, it should be emphasized that in practice, the implementation of a uniform wall configuration may be technically difficult. In all cases, the main operability bottleneck is the hot utilities availability. The designs arising from optimization are characterized by small tube diameters and long heat exchangers. Control schemes based only on the heating medium inlet temperature were proved to be not economical, and control only by the heating medium flow rate seems to be ineffective.

Beyond these practical conclusions the work also demonstrates the increasing ability to solve dynamic optimization problems involving fairly large models with many interacting decisions and constraints. There has been no previous work in the literature to our knowledge that addresses this problem in such an integrated way and with this degree of detail. Moreover, the importance of using accurate dynamic models for this study cannot be overemphasized. This is especially true given the tight process economics and the tight operating constraints under which a pasteurizer operates.

The main problem encountered was the extremely large computational cost for the solution of the optimization problems. This is due to the high complexity and the nature of the models described by integral, partial differential, and algebraic equations. A large proportion of the time was spent on sensitivity integration—as much as 30 times that of the state integration only. However, current computer developments toward parallel architectures will provide a way to dramatically reduce such computational costs in the near future. Finally, it is worth pointing out that, because of the use of local optimization algorithms for the solution of the NLP, the global optimality of any solution obtained with our approach cannot normally be guaranteed. This is a common deficiency of optimization-based design methods that can only be overcome by the adoption of global optimization techniques.

Notation

CC = total annual operating cost, \$/yr
C_{ps} = heating medium specific heat capacity, J/kg·K
$C_{p,\mathrm{milk}}$ = specific heat capacity of milk, J/kg·K
d = heat exchanger diameter, m
l_k = duration of control interval k
$l_k^{\max}$ = maximum duration of control interval k
$l_k^{\min}$ = minimum duration of control interval k
L = heat exchanger length, m
m_{steam} = steam flow rate for the heating operation, kg/s
x = fraction of the deposit which is removable
y = fraction of the deposit which is not removable
δ = deposit thickness
ζ = normalized axial domain

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Appendix: Simulation of Cleaning Operation

The cleaning operation is simulated using the model of Bird and Fryer (1992). This model includes two simple equations. The deposit of thickness δ is described by two layers, an upper swelled-deposit layer of thickness $x\delta$ that can be removed and a lower layer of thickness $y\delta$ which is not yet removable at time t. The equations governing the rate of change of thickness of the two layers are simply expressed as:

$$\frac{d(y\delta)}{dt} = -k_y \tag{A1}$$

$$\frac{d(x\delta)}{dt} = k_y - k_x x\delta \tag{A2}$$

The constants k_x and k_y are given as a function of temperature by Bird and Fryer (1992). In our simulation, the cleaning solution concentration is 1 wt. % sodium hydroxide at a velocity of 0.174 m/s and temperature 60°C. Under these conditions (which are the optimal according to Bird and Fryer) the aforementioned rate constants are:

$$k_x = 0.00673 \quad \text{m/s} \tag{A3}$$

$$k_y = 8.315 \times 10^{-7} \quad \text{s}^{-1} \tag{A4}$$

The cleaning operation is simulated in the *gPROMS* modeling system (Oh and Pantelides, 1996) where the initial deposit thickness is taken from the simulation of the heating operation. As expected, different heating operations lead to different cleaning times. The cleaning time is defined as the time required for the deposit thickness to reach a target low value (taken here to be 10^{-9} m).

Cleaning time can be related to heating time on the basis of the simulation results. This allows the cleaning cost for the objective function used in the dynamic optimization problem to be accounted for. Assuming a cubic variation between the cleaning and heating time the following correlation can be used.

$$t_{\text{clean}} = 0.353 + 0.01237 \cdot t_{\text{heat}} - 2.55 \times 10^{-4} \cdot t_{\text{heat}}^2 + 1.87 \times 10^{-6} \cdot t_{\text{heat}}^3 \quad h \tag{A5}$$

where t_{heat} and t_{clean} are the heating and cleaning times, respectively.

In principle, one could explicitly integrate Eqs. A1 and A2 within the dynamic optimization. However, this leads to a multistage optimal control problem that is very difficult to solve using currently available algorithms.

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